Natural Resolution of the Proton Size Puzzle

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We show that off-mass-shell effects arising from the internal structure of the proton provide a new proton polarization mechanism in the Lamb shift, proportional to the lepton mass to the fourth power. This effect is capable of resolving the current puzzle regarding the difference in the proton radius extracted from muonic compared with electronic hydrogen experiments. These off-mass-shell effects could be probed in several other experiments.

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The recent, extremely precise extraction of the proton radius [1] from the measured energy difference between the $2P_{3/2}^{F=2}$ and $2S_{1/2}^{F=1}$ states of muonic hydrogen (H) has created considerable interest. Their analysis yields a proton radius that is smaller than the CODATA [2] value (extracted mainly from electronic H) by about 4% or 5.0 standard deviations. This implies [1] that either the Rydberg constant has to be shifted by 4.9 standard deviations or that the QED calculations for hydrogen are insufficient. Since the Rydberg constant is extremely well measured, and the QED calculations seem to be very extensive and highly accurate, the muonic H finding presents a significant puzzle to the entire physics community.

Our analysis is motivated by the fact that muonic hydrogen is far smaller than electronic hydrogen and therefore more sensitive to corrections arising from hadron structure. In particular, we consider the lowest order correction associated with off-shell behaviour at the photon-nucleon vertex, showing that it can very naturally account for the difference reported by Pohl *et al.*. Since at the present state of development of hadronic physics it is not possible to provide a precise value for this correction, our result may be viewed as a phenomenological study of the sensitivity of muonic hydrogen to important aspects of proton structure. It should spur further study of processes which could be sensitive to off-shell changes in proton structure. In alternate language, the explanation which we present may be viewed as a new contribution from proton polarization that is not constrained by dispersion relations but which can be studied in systems other than the hydrogen atom.

We begin with a brief discussion of the relevant phenomenology. Pohl *et al.* show that the energy difference between the $2P_{3/2}^{F=2}$ and $2S_{1/2}^{F=1}$ states, $\Delta\widetilde{E}$ is given by

$$\Delta\widetilde{E} = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV}, \quad (1)$$

where r_p is given in units of fm. Each of the three coefficients is obtained from extensive theoretical work [3–7], typically confirmed by several groups. Studies of the relevant atomic structure calculations and corresponding efforts to improve those have revealed no variations large enough to significantly affect the above equation [8, 9]. Using this equation, we see that the difference between the Pohl and CODATA values of the proton radius would be entirely removed by an increase of the first term on the rhs of Eq. (1) by just 0.31 meV=3.1×10^{-10} MeV, but an effect of even half that much would be large enough to dissipate the puzzle. Finding a new effect of about that value resolves the puzzle provided that the corresponding effect in electronic H is no more than a few parts in a million (the current difference between theory and experiment [3]). An effect that gives a contribution to $\Delta\widetilde{E}$ of the form $\alpha^5 \frac{m^4}{M^3}$ (with m the lepton mass and M the proton mass) could therefore resolve the proton radius puzzle and cause no disagreement in electronic H.

The search to find such an effect has attracted considerable interest. New physics beyond the Standard Model must satisfy a variety of low-energy constraints and so far no explanation of the proton radius puzzle has been found that satisfies these constraints [10–14]. Attention has been paid to the third term of Eq. (1) [15], with the result that its current uncertainties are far too small to resolve the proton radius puzzle [16, 17].

We therefore seek an explanation based on the fact that the proton is not an elementary Dirac particle, and that many features of its interactions are still unknown. In particular, consider the electromagnetic vertex function which must depend on all of the relevant invariants. For a proton of initial four-momentum p, the most general expression must include a term, dependent on the proton virtuality, that is proportional to $p^2 - M^2$ and/or $p\cdot\gamma_N - M$, where the subscript N denotes acting on a nucleon, and M is the nucleon mass. Such terms have been discussed for a very long time in atomic [6, 7] and nuclear physics [18]-[32]. They have been of special concern in relation to the difference between free and bound deep inelastic structure functions measured in the EMC effect [18]-[22], nucleon-nucleon scattering [23] and electromagnetic interactions involving nucleons [24, 25], notably quasi-elastic scattering [26]-[32].

Many possible forms [24, 25] include the effects of proton virtuality; we consider three that could be significant for the Lamb shift. We write the Dirac part of the vertex function for a proton of momentum p to absorb a photon

of momentum $q = p' - p$.as:

$$\Gamma^\mu(p',p) = \gamma_N^\mu F_1(-q^2) + F_1(-q^2)F(-q^2)\mathcal{O}^\mu_{a,b,c} \tag{2}$$
$$\mathcal{O}_a^\mu = \frac{(p+p')^\mu}{2M}[\Lambda_+(p')\frac{(p\cdot\gamma_N - M)}{M} + \frac{(p'\cdot\gamma_N - M)}{M}\Lambda_+(p)]$$
$$\mathcal{O}_b^\mu = ((p^2 - M^2)/M^2 + (p'^2 - M^2)/M^2)\gamma_N^\mu$$

$$\mathcal{O}_c^\mu = \Lambda_+(p')\gamma_N^\mu \frac{(p\cdot\gamma_N - M)}{M} + \frac{(p'\cdot\gamma_N - M)}{M}\gamma_N^\mu \Lambda_+(p),$$

where three possible forms are displayed. Other terms of the vertex function needed to satisfy the WT identity do not contribute significantly to the Lamb shift and are not shown explicitly. The proton Dirac form factor, $F_1(-q^2)$ is empirically well represented as a dipole $F_1(-q^2) = (1 - q^2/\Lambda^2)^{-2}$, ($\Lambda = 840$ MeV) for the values of $-q^2 \equiv Q^2 > 0$ of up to about 1 GeV^2 needed here. $F(-q^2)$ is an off-shell form factor, and $\Lambda_+(p) = (p\cdot\gamma_N + M)/(2M)$ is an operator that projects on the on-mass-shell proton state. We use $\mathcal{O}_a$ unless otherwise stated.

We take the off-shell form factor $F(-q^2)$ to vanish at $q^2 = 0$. This means that the charge of the off-shell proton will be the same as the charge of a free proton, and is demanded by current conservation as expressed through the Ward-Takahashi identity [24, 25]. We assume

$$F(-q^2) = \frac{-\lambda q^2/b^2}{(1 - q^2/\widetilde{\Lambda}^2)^{1+\xi}}. \tag{3}$$

This purely phenomenological form is simple and clearly not unique. The parameter b is expected to be of the order of the pion mass, because these longest range components of the nucleon are least bound and more susceptible to the external perturbations putting the nucleon off its mass shell. At large values of $|q^2|$, F has the same fall-off as F_1, if $\xi = 0$. We take $\widetilde{\Lambda} = \Lambda$ here.

We briefly discuss the expected influence of using Eq. (2). The ratio, R, of off-shell effects to on-shell effects, $R \sim \frac{(p\cdot\gamma_N - M)}{M}\lambda\frac{q^2}{b^2}$, ($|q^2| \ll \Lambda^2$) is constrained by a variety of nuclear phenomena such as the EMC effect (10-15%), uncertainties in quasi-elastic electron-nuclear scattering [26], and deviations from the Coulomb sum rule [27]. For a nucleon experiencing a 50 MeV central potential, $(p\cdot\gamma_N - M)/M \sim 0.05$, so $\lambda q^2/b^2$ is of order 2. The nucleon wave functions of light-front quark-models [33] contain a propagator depending on M^2. Thus the effect of nucleon virtuality is proportional to the derivative of the propagator with respect to M, or of the order of the wave function divided by difference between quark kinetic energy and M. This is about three times the average momentum of a quark (~ 200 MeV/c) divided by the nucleon radius or roughly $M/2$. Thus $R \sim (p\cdot\gamma_N - M)2/M$, and the natural value of $\lambda q^2/b^2$ is of order 2.

The lowest order term in which the nucleon is sufficiently off-shell in a muonic atom for this correction to produce a significant effect is the two-photon exchange diagram of Fig. 1 and its crossed partner, including an

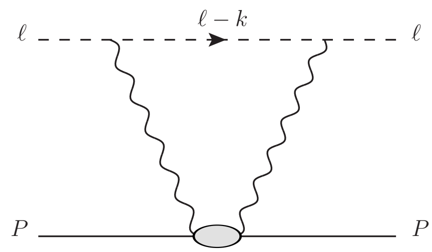

FIG. 1: Direct two-photon exchange graph corresponding to the hitherto neglected term. The dashed line denotes the lepton; the solid line, the nucleon; the wavy lines photons; and the ellipse the off-shell nucleon.

interference between one on-shell and one off-shell part of the vertex function. The change in the invariant amplitude, $\mathcal{M}_{\rm Off}$, due to using Eq. (2) along with $\mathcal{O}_a^\mu$, to be evaluated between fermion spinors, is given in the rest frame by

$$\mathcal{M}_{\rm Off} = \frac{e^4}{2M^2}\int \frac{d^4k}{(2\pi)^4}\frac{F_1^2(-k^2)F(-k^2)}{(k^2+i\epsilon)^2} \tag{4}$$
$$\times(\gamma_N^\mu(2p+k)^\nu + \gamma_N^\nu(2p+k)^\mu)$$
$$\times\left[\gamma_\mu\frac{(l\cdot\gamma - k\cdot\gamma + m)}{k^2 - 2l\cdot k + i\epsilon}\gamma_\nu + \gamma_\nu\frac{(l\cdot\gamma + k\cdot\gamma + m)}{k^2 + 2l\cdot k + i\epsilon}\gamma_\mu\right],$$

where the lepton momentum is $l = (m,0,0,0)$, the virtual photon momentum is k and the nucleon momentum $p = (M,0,0,0)$. The intermediate proton propagator is cancelled by the off-mass-shell terms of Eq. (2). This graph can be thought of as involving a contact interaction and the amplitude in Eq. (4) as a new proton polarization correction corresponding to a subtraction term in the dispersion relation for the two-photon exchange diagram that is not constrained by the cross section data [34]. The resulting virtual-photon-proton Compton scattering amplitude, containing the operator $\gamma_N^\mu\gamma_N^\nu$ corresponds to the T_2 term of conventional notation [35], [36]. Eq. (4) is gauge-invariant; not changed by adding a term of the form $k^\mu\, k^\nu/k^4$ to the photon propagator.

Evaluation proceeds in a standard way by taking the sum over Dirac indices, performing the integral over k^0 by contour rotation, $k^0 \to -ik^0$, and integrating over the angular variables. The matrix element $\mathcal{M}$ is well approximated by a constant in momentum space, for momenta typical of a muonic atom, and the corresponding potential $V = i\mathcal{M}$ has the form $V(\mathbf{r}) = V_0\delta(\mathbf{r})$ in coordinate space. This is the "scattering approximation" [3]. Then the relevant matrix elements have the form $V_0\left|\Psi_{2S}(0)\right|^2$, where Ψ_{2S} is the muonic hydrogen wave function of the state relevant to the experiment of Pohl *et al.* We use $\left|\Psi_{2S}(0)\right|^2 = (\alpha m_r)^3/(8\pi)$, with the lepton-proton re-

duced mass, m_r. The result

$$\langle 2S|V|2S\rangle = \frac{-\alpha^5 m_r^3}{M^2}\frac{8}{\pi}\lambda\frac{mM}{b^2}F_L(m) ,$$
$$F_L(m) \equiv \frac{1}{\beta}\int_0^\infty \frac{\sqrt{(x+\beta)/x}-1}{(1+x^2)^{5+\xi}}\, xdx , \quad (5)$$

(where $\beta \equiv 4m^2/\Lambda^2$) shows a new contribution to the Lamb shift, proportional to m^4 and therefore negligible for electronic hydrogen. Using $\mathcal{O}_a^\mu$ leads to a vanishing hyperfine HFS splitting because the operator γ_N^μ is odd unless $\mu = 0$.

We next seek values of the model parameters λ, b, ξ of Eq. (3), chosen to reproduce the value of the needed energy shift of 0.31 meV with a value of λ of order unity. Numerical evaluation, using $\xi = 0$, $\widetilde{\Lambda} = \Lambda$, shows that

$$\frac{\lambda}{b^2} = \frac{2}{(79\text{MeV})^2} \quad (6)$$

leads to 0.31 meV. If ξ is changed substantially from 0 to 1 the required value of λwould be increased by about 10%. If our mechanism increases the muonic Lamb shift by 0.31 meV, the change in the electronic H Lamb shift for the 2S-state is about 9 Hz, significantly below the current uncertainty in both theory and experiment [3]. Should some other effect account for part of the proton radius puzzle, the value of λ/b^2 would be decreased. We also caution that other systems in which one might aim to test this effect could show sensitivity to the value of ξ or $\tilde{\Lambda}$ in Eq. (3).

The other operators appearing in Eq. (2) yield similar results when used to evaluate $\mathcal{M}_{\text{off}}$. Using $\mathcal{O}_b$, gives a term of the T_2 form with a Lamb shift twice that of $\mathcal{O}_a$, and also a HFS term that is about -1/6 of its Lamb shift, so the value of λ/b^2 would be decreased by 3/5. The use of $\mathcal{O}_c$, gives a term of the T_1 form and the same Lamb shift as $\mathcal{O}_a$, as well as a HFS term that is -1.7 times its Lamb shift. In this case, the value of λ/b^2 would be about - 3/2 times that of Eq. (6). The HFSs are small enough to be well within current experimental and theoretical limits for electronic hydrogen. Thus each operator leads to a reasonable explanation of the proton radius puzzle.

It is necessary to comment on the difference between our approach, which yields a relevant proton polarization effect, and others [36], [37] which do not. The latter use a current-conserving representation of the virtual-photon proton scattering amplitude in terms of two unmeasurable scalar functions, $T_{1,2}$. Dispersion relations are used to relate $T_{1,2}$ to their measured imaginary parts. However, terms with intermediate nucleon states are treated by evaluating Feynman diagrams. This allows the removal of an infrared divergence by subtracting the first iteration of the effective potential that appears in the wave function. But the Feynman diagrams involve off-shell nucleons, so that their evaluation for composite particles must be ambiguous. For example, using two different forms of the on-shell electromagnetic vertex function, related by using the Gordon identity, leads to results that differ. This ambiguity in obtaining $T_{1,2}$ is removed in our approach by postulating Eq. (2) and evaluating its consequences. Note also that in order to evaluate the term involving T_1 using a dispersion relation one must introduce a subtraction function, $T_1(0, q^2)$. This is unconstrained by prior data [35] because the value of σ_L/σ_T at $\nu = \infty$ is not determined [38]. Pachucki [36], in Eq.(31), assumes a form proportional to q^2 (see our Eq. (3)) times the very small proton magnetic polarizability. However we are aware of no published derivation of this result.

In conclusion, we have shown that a simple off-shell correction to the photon-proton vertex, which arises naturally in quantum field theory and is of natural size and consistent with gauge invariance, is capable of resolving the discrepancy between the extraction of the proton charge radius from Lamb shift measurements in muonic and electronic hydrogen. Off-shell effects of the proton form factor were an explicit concern of both Zemach [6] and Grotch & Yennie [7]. However, it is only with the remarkable improvement in experimental precision recently achieved [1] that it has become of practical importance. Within the field of nuclear physics there is great interest in the role that the modification of nucleon structure in-medium may play in nuclear structure [39, 40]. We stress that the effect postulated here can be investigated in lepton-nucleus scattering via the binding effects of the nucleon, as well as by lepton-proton scattering in arenas where two photon (or γ, Z) effects are relevant.

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